Calculus

Diocese of Greensburg Curriculum

Unit	Standards	Content	Skills

Limits	**CA: CCSS: Mathematics**
	CA: Advanced Mathematics
	Calculus
	Calculus: When taught in high school, calculus should be presented with the same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the following content in a typical academic year. For example, some districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject.
	
	1.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity: 1.1 Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of
		• Finding Limits Numerically
		• Finding Limits Graphically
		• Finding Limits Analytically
		  ○ Squeeze Theorem
		  ○ Absolute Value
		  ○ One-Sided
		• Continuity
		• Infinite Limits
		• Limits at Infinity
		  ○ Horizontal Asymptotes
		  ○ Vertical Asymptotes
		  ○ Slant Asymptotes
			Knowledge
			• Use the concept of limit serves to describe/predict a function's behavior near (not at) x = a
			• Determine limits that exist at x = a even though the function is not defined there
			• Use the Limit Theorems to compute limits
			• Know the 3 ways that a function might not be continuous (hole, jump, vertical asymptote)
			
			Comprehension
			• Use certain "obvious" limits to find more complex limits
			
			Application
			• Apply the fact that a limit exists and equals L if and only if both its left- and right-hand limits equal L
			• Apply the EVT to find absolute maximum and minimum values for a function on a closed interval
			• Use the definition of continuity to evaluate limits of continuous functions and to determine continuity
			Use limits to remove removable singularities

	functions. 1.2 Students use graphical calculators to verify and estimate limits. 1.3 Students prove and use special limits, such as the limits of (sin(x))/x and (1-cos(x))/x as x tends to 0. 2.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.		**Analysis** • Identify approaching ±infinity **Synthesis** • Apply the fact that combinations of continuous functions are continuous **Evaluation** • Assess incorrect work to find errors.
Derivatives	**CA: CCSS: Mathematics** **CA: Advanced Mathematics** **Calculus** **Calculus: When taught in high school, calculus should be presented with the same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the following content in a typical academic year. For example, some districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this**	• Limit Definition of the Derivative • Polynomials • Trigonometric Functions • Product Rule • Quotient Rule • Chain Rule • Implicit Differentiation • Logarithmic Functions • Logarithmic Method • Exponential Functions • Inverse Trigonometric Functions • Hyperbolic Trigonometric Functions	**Knowledge** • Define a derivative • Describe a derivative • Identify which rules are needed in which scenarios **Comprehension** • Relate derivatives to functions • Restate the definition of a derivative **Application** • Modify previous rules to create new rules • Produce equations of tangent lines on functions **Analysis**

	content will be exposed to both aspects of the subject. 4.0 Students demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability: 4.1 Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function. 4.2 Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function. 4.3 Students understand the relation between differentiability and continuity. 4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions. 5.0 Students know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions. 7.0 Students compute derivatives of higher orders. 9.0 Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.		• Analyze functions for patterns in taking the derivative **Synthesis** • Combine different rules into larger, more complex problems **Evaluation** • Assess incorrect work to find errors
Applications of Derivatives	**CA: CCSS: Mathematics** **CA: Advanced Mathematics** **Calculus** **Calculus: When taught in high school, calculus should be presented with the**	• Differentials and Newton's Method • Graph Sketching ○ First Derivative Test	**Knowledge** • Understand how functions can cross their horizontal asymptote

	same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the following content in a typical academic year. For example, some districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject. 3.0 Students demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem. 6.0 Students find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth. 8.0 Students know and can apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule. 10.0 Students know Newton's method for approximating the zeros of a function. 11.0 Students use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.	○ Second Derivative Test ○ Asymptotes • Optimization • Related Rates • L'Hopital's Rule	• Use knowledge about asymptotes to help graph functions • Use knowledge about functions behavior to help find asymptotes **Comprehension** • Describe what f and f ' say about each other graphically and numerically • Graphically find where f is rising/falling by knowing where f ' is positive/negative • Graphically find where f is concave up/down by knowing where f " is positive/negative **Application** • Apply the fact that f ' tells how fast f increases or decreases • Apply the IVT • Find stationary points, local and global extrema, and inflection points • Find inflection points from sign changes in the second derivative • Find extrema via the First and Second Derivative Tests • Find horizontal and vertical asymptotes • Compute limit of f as x approaches positive and negative infinity and using the result to find horizontal asymptotes • Find the equation of vertical and horizontal asymptotes for a function

	12.0 Students use differentiation to solve related rate problems in a variety of pure and applied contexts.		**Analysis** • Convert some indeterminate forms into the form required by L'Hopital's Rule • Use algebra and L'Hopital's rule to evaluate indeterminate limits **Synthesis** • Graphically find where f is concave up/down by knowing where f ' is increasing/decreasing **Evaluation** • Students will assess incorrect work to find errors.

Integration	**CA: CCSS: Mathematics**
	CA: Advanced Mathematics
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		• Reimann Sums
		• Indefinite Integration
		  ○ Movement
		  ○ Trigonometric Functions
		  ○ Logarithmic Functions
		  ○ Exponential Functions
		• Definite Integrals
		• Fundamental Theorems of Calculus
		• Integration by Substitution (u-substitution)
		• Integration by Parts (inverse of the product rule)
		• Additional Techniques
			Knowledge
			
			• Construct left, right, and midpoint rectangles and trapezoids to approximate a given definite integral
			• Use the height of an approximating rectangle as the function's value at the sampling point for that subinterval
			• Interpret the Integral as an area function of x
			• Compute $\int f(x)dx$ algebraically
			
			Comprehension
			
			• Compute the integral using geometric formulas or symmetry properties

	Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject. 13.0 Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals. 14.0 Students apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals. 15.0 Students demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives. 16.0 Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work. 17.0 Students compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate. 18.0 Students know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals. 19.0 Students compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the	○ Function Manipulation ○ Trigonometric Substitution • Area Between Curves	• Calculate delta x as the width of each subinterval and base of each approximating rectangle approximation improves **Application** • Apply the properties of integrals to aid in computing Integrals • Apply the sum and constant multiple rules apply • Apply the fact that, as the number of equal subintervals grows, delta x approaches 0 and accuracy of • Use the Fundamental Theorem of Calculus to find specific areas **Analysis** • Use substitutions to find antiderivatives • Partition (subdivide) a region's interval into n equal subintervals for approximation **Synthesis** • Choose the sampling point for each subinterval to get left, right, or midpoint sums **Evaluation** • Students will evaluate incorrect work for errors.

	algebraic techniques of partial fractions and completing the square. 20.0 Students compute the integrals of trigonometric functions by using the techniques noted above. 21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.		